A Kinematic Saturation Bound on Gravitational Acceleration

Amar Arun Naik
Independent Researcher, Pune, Maharashtra, India
Email: theamarnaik@gmail.com

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Abstract

We propose a minimal covariant kinematic bound on gravitational acceleration by restricting the admissible spacetime congruences in strong-field regimes. The construction introduces no new dynamical fields and does not modify Einstein's field equations. The bound is formulated directly in terms of the four-acceleration of timelike congruences and is shown explicitly to yield finite acceleration and curvature in the Schwarzschild geometry when expressed in Painlevé–Gullstrand coordinates. The weak-field limit recovers Newtonian gravity. The result provides a compact and physically transparent phenomenological constraint on strong gravitational dynamics.

1 Introduction and Kinematic Saturation Postulate

Consider a timelike unit four-velocity field u^μ representing an admissible spacetime congruence, satisfying

$$u^\mu u_\mu = -1. \tag{1}$$

The associated four-acceleration is defined covariantly as

$$a^\mu := u^\nu \nabla_\nu u^\mu. \tag{2}$$

We impose the following kinematic admissibility condition:

$$a^\mu a_\mu \leq a_{\max}^2, \qquad a_{\max} \equiv \frac{c^4}{4GM}, \tag{3}$$

where M is the gravitating mass sourcing the geometry and G and c are the gravitational constant and invariant speed, respectively.

This postulate restricts the magnitude of proper acceleration attainable by any physical timelike congruence without modifying the spacetime field equations. It acts as a phenomenological constraint excluding unphysical divergences in strong-field regimes.

2 Schwarzschild Geometry in Painlevé–Gullstrand Coordinates

Throughout this section, we work in units with $c = 1$ for clarity, restoring factors of c when interpreting physical bounds.

The Schwarzschild spacetime in Painlevé–Gullstrand coordinates is given by

$$ds^2 = -dt^2 + \left(dr + \sqrt{\frac{2GM}{r}}\,dt\right)^2 + r^2 d\Omega^2. \tag{4}$$

The associated radial inflow velocity of space is

$$v(r) = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{r_s}{r}}, \tag{5}$$

where $r_s = 2GM$ is the Schwarzschild radius. The inflow velocity asymptotically saturates at the invariant speed as $r \to r_s^+$.

The natural infalling ("raindrop") congruence is described by

$$u^\mu = \left(\frac{dt}{d\tau}, \frac{dr}{d\tau}, 0, 0\right) = \left(1, -\sqrt{\frac{2GM}{r}}, 0, 0\right). \tag{6}$$

Since the proper acceleration is defined covariantly by $a^\mu = u^\nu \nabla_\nu u^\mu \sim dv/d\tau$, saturation of the inflow velocity at the invariant speed directly enforces a bound on attainable acceleration.

A direct computation yields the invariant proper acceleration magnitude

$$a(r) := \sqrt{a^\mu a_\mu} = \frac{GM}{r^2}\sqrt{1 - \frac{r_s}{r}}. \tag{7}$$

As $r \to r_s^+$, one finds

$$\lim_{r \to r_s^+} a(r) = 0, \tag{8}$$

so that the acceleration remains finite and bounded everywhere outside the horizon, consistent with the kinematic bound (3).

3 Bounded Curvature Interpretation

The Kretschmann scalar for the Schwarzschild geometry is

$$K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{48G^2M^2}{r^6}. \tag{9}$$

If admissible physical congruences are restricted by the saturation bound and cannot probe $r < r_s$, the curvature is bounded by

$$K_{\max} = \frac{48G^2M^2}{r_s^6} = \frac{3}{4}\frac{c^8}{G^4M^4}, \tag{10}$$

which is finite. Classical curvature divergences are therefore rendered operationally inaccessible at the level of admissible kinematics rather than through modification of the field equations.

4 Weak-Field Limit

For $r \gg r_s$, the proper acceleration admits the expansion

$$a(r) = \frac{GM}{r^2}\left(1 - \frac{1}{2}\frac{r_s}{r} + \mathcal{O}\left(\frac{r_s^2}{r^2}\right)\right), \tag{11}$$

recovering the Newtonian gravitational acceleration at leading order.

5 Discussion and Conclusion

We have presented a compact and fully covariant kinematic saturation bound on gravitational acceleration. The construction leaves Einstein's field equations unchanged and introduces no new degrees of freedom, acting instead as an admissibility constraint on timelike observers. In the Schwarzschild geometry, the bound yields finite proper acceleration and renders classical singularities operationally inaccessible while preserving the correct weak-field behavior. Owing to its minimal and equation-driven formulation, the approach may serve as a useful phenomenological constraint in the analysis of strong gravitational fields.

References

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